Q1a

a) rewrite all the functions as powers of 2

$\log_2 2^6 + 3\log_2 2^4 - 2\log_2 2^5$

rewrite powers as coefficients (or coefficients as powers!)

$6\log_2 2 + 3(4)\log_2 2 - 2(5)\log_2 2$

$(6 + 12 - 10)\log_2 2$

$8\log_2 2 = 8(1) =$ **8**

Q1b

b) rewrite coefficients as powers

$\ln 2^3 + \ln 5^2 - \ln\sqrt{10000}$

$\sqrt{10^4} = 10^{\frac{4}{2}} = 10^2$

$(2\times 5)^2$

$\ln 2^3 + \ln 5^2 - \ln(2^2 5^2)$

Use log laws of addition & subtraction to combine

$\ln \frac{2^3 5^2}{2^2 5^2} =$ **$\ln 2$**

Q2a

a) rewrite 16 as a power of 4

$$4^{3x+2} = (4^2)^{x+6}$$
$$= 4^{2x+12}$$

equate powers

$$3x+2 = 2x+12$$

$$\boxed{x = 10}$$

Q2b

b) $4^{2x+3} = 100$

$$\log_4 4^{2x+3} = \log_4 100$$

$$2x+3 = \log_4 100$$

$$x = \frac{(\log_4 100) - 3}{2}$$

$$= \boxed{0.161} \quad (3sf)$$

Q3a

a) $e^{3x-2} = 3$

$$\ln e^{3x-2} = \ln 3$$

$$3x-2 = \ln 3$$

$$\boxed{x = \frac{2+\ln 3}{3}}$$

Q3b

b) $3(e^x)^2 - 14(e^x) + 8 = 0$ ← quadratic in e^x!

let $e^x = y$

$3y^2 - 14y + 8 = 0$

$\times 24$
$+ -14$
$-12, -2$

$3y^2 - 2y - 12y + 8$

$(y-4)(3y-2) = 0$

$y = 4 \quad y = \frac{2}{3}$

$e^x = 4 \quad e^x = \frac{2}{3}$

$x = \ln 4, \ln \frac{2}{3}$

Q4a

rewrite 9 as a power of 3

a) Using law ①

$\ln(3^4)^2 + \ln 3^3 - \ln 3^2$

$\ln 3^8 + \ln 3^3 - \ln 3^2$

Using laws ② & ③

$\ln \frac{3^8 3^3}{3^2} = \ln 3^9$

$= \boxed{9\ln 3}$

① $k\log_a x = \log_a x^k$

② $\log_a xy = \log_a x + \log_a y$

③ $\log_a \frac{x}{y} = \log_a x - \log_a y$

Q4b

b) rewrite using ①

$\log_a x^2 + \log_a (x+1)^3 - \log_a 4(x+2)$

combine using ② & ③

$= \boxed{\log_a \left(\frac{x^2(x+1)^3}{4(x+2)} \right)}$

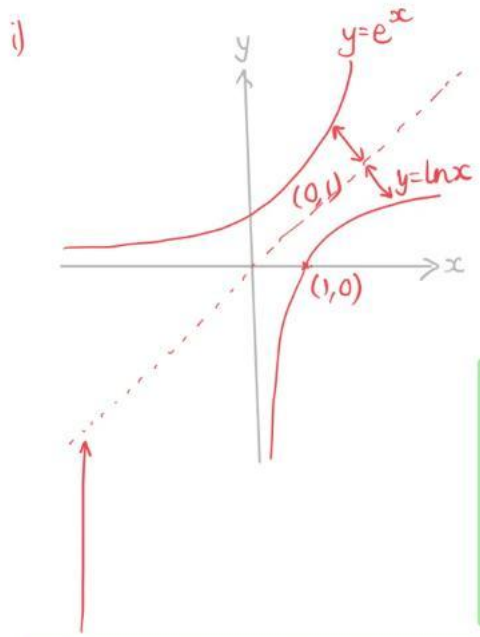
① $k\log_a x = \log_a x^k$

② $\log_a xy = \log_a x + \log_a y$

③ $\log_a \frac{x}{y} = \log_a x - \log_a y$

Q5

i)



x	-1	1	2	3
$\ln x$	X	0	0.7	1.1

math error +0.7 +0.4

as x increases, $\ln x$ increases at a decreasing rate

asymptote for $y=e^x$ is $y=0$

asymptote for $y=\ln x$ is $x=0$

ii) $x=y$ (line of reflection)

Q6

$(5^x)^2 - 8(5^x) + 12 = 0$

quadratic in 5^x!

Let $y = 5^x$

$y^2 - 8y + 12 = 0$

$\times 12$, $+ -8$, $-6, -2$

$(y-6)(y-2)=0$

$y=6 \quad y=2$

$5^x = 6 \quad 5^x = 2$

$x = \log_5 6, \log_5 2$

Q7

$$3^{x-1} = \frac{6^{2x}}{6} = 6^{2x-1}$$

$$= (2\times 3)^{2x-1}$$

$$3^{x-1} = 2^{2x-1}\,3^{2x-1}$$

$$3^{(x-1)-(2x-1)} = 2^{2x-1}$$

$$3^{-x} = 2^{2x-1}$$

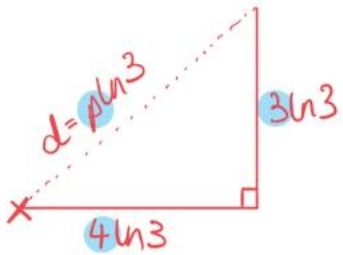
$$\ln 3^{-x} = \ln 2^{2x-1}$$

$$-x\ln 3 = (2x-1)\ln 2$$

$$x = \frac{1}{2+\frac{\ln 3}{\ln 2}} \times \frac{\ln 2}{\ln 2}$$

$$x = \frac{\ln 2}{2\ln 2+\ln 3} = \frac{\ln 2}{\ln 4+\ln 3} = \boxed{\frac{\ln 2}{\ln 12}}$$

Q8



Spot pythagorus' 3-4-5 triangle scaled by $\ln 3$ $\therefore$ $p = 5$!

distance: $\boxed{5\ln 3}$

OR Alternatively use pythagorus' theorum to find the length of the hypothenuse...

$$d = \sqrt{(4\ln 3)^2 + (3\ln 3)^2} \qquad \boxed{d = 5\ln 3}$$

Q9

$5\ln 2 + 5\ln e$ → $\ln e = 1$

$= 5(\ln 2 + \ln e)$

Combine using log law: $\log_a xy = \log_a x + \log_a y$

$= \boxed{5\ln 2e}$

Q10

$\log_3(x+4) - 2\log_3 x = 4$

① $k\log_a x = \log_a x^k$

② $\log_a \frac{x}{y} = \log_a x - \log_a y$

Apply law ①

$\log_3(x+4) - \log_3 x^2 = 4$

Apply law ②

$\log_3 \frac{(x+4)}{x^2} = 4$

$3^4 = \frac{x+4}{x^2}$

$81x^2 - x - 4 = 0$

$x = 0.228, -0.216$ (3sf)

reject negative

$\boxed{x = 0.228}$ (3sf)

Q11

$k \log_a x = \log_a x^k$

$\log_x (x+2)^2 = 3$

$x^3 = (x+2)^2$

$x^3 = x^2 + 4x + 4$

$x^3 - x^2 - 4x - 4 = 0$

$x = 2.88$ (3sf)